THE MEAN

Population: $\mu = \frac{\Sigma X}{N}$ Sample: $M = \frac{\Sigma X}{n}$

SUM OF SQUARES

Definitional: $SS = \Sigma(X - \mu)^2$

Computational: $SS = \Sigma X^2 - \frac{(\Sigma X)^2}{N}$

VARIANCE

Population: $\sigma^2 = \frac{SS}{N}$ Sample: $s^2 = \frac{SS}{n-1}$

STANDARD DEVIATION

Population: $\sigma = \sqrt{\frac{SS}{N}}$ Sample: $s = \sqrt{\frac{SS}{n-1}}$

***z*-SCORE (FOR LOCATING AN *X* VALUE)**

$z = \frac{X - \mu}{\sigma}$

BINOMIAL (NORMAL APPROXIMATION)

$z = \frac{X - pn}{\sqrt{npq}}$ or $z = \frac{X/n - p}{\sqrt{pq/n}}$

***z*-SCORE (FOR LOCATING A SAMPLE MEAN)**

$z = \frac{M - \mu}{\sigma_M}$ where $\sigma_M = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{\sigma^2}{n}}$

when z is known but M isn't $M = z(\sigma_m) + \mu$

***t* STATISTIC (SINGLE SAMPLE)**

$t = \frac{M - \mu}{s_M}$ where $s_M = \sqrt{\frac{s^2}{n}}$

***t* STATISTIC (INDEPENDENT MEASURES)**

$t = \frac{(M_1 - M_2) - (\mu_1 - \mu_2)}{s_{(M_1 - M_2)}}$

#3. where $s_{(M_1 - M_2)} = \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$ and $s_p^2 = \frac{SS_1 + SS_2}{df_1 + df_2}$

#1. $s(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{SS_1 + SS_2}{n_1 + n_2 - 2} \cdot \left(\frac{n_1 + n_2}{n_1 \cdot n_2}\right)}$

***t* STATISTIC (RELATED SAMPLES)**

$t = \frac{M_D - \mu_D}{s_{M_D}}$ where $s_{M_D} = \sqrt{\frac{s^2}{n}}$

ESTIMATION

t Statistic (Single Sample)

$\mu = M \pm t s_M$

t Statistic (Independent Measures)

$\mu_1 - \mu_2 = M_1 - M_2 \pm t s_{(M_1 - M_2)}$

t Statistic (Related Samples)

$\mu_D = M_D \pm t s_{M_D}$

- for sample size

$d = \frac{\mu_{true} - \mu_{hypo}}{\sigma}$

#2 $s(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$ • cont.

cont under the √ don't divide $\cdot \left(\frac{n_1 + n_2}{n_1 \cdot n_2}\right)$

INDEPENDENT-MEASURES ANOVA

$SS_{total} = \Sigma X^2 - \frac{G^2}{N}$ $df_{total} = N - 1$

$SS_{between} = \Sigma \frac{T^2}{n} - \frac{G^2}{N}$ $df_{between} = k - 1$

$SS_{within} = \Sigma SS_{\text{inside each treatment}}$ $df_{within} = N - k$

$F = \frac{MS_{between}}{MS_{within}}$ where each $MS = \frac{SS}{df}$

Hypothesis w/ Variances $H_0: \sigma^2 = a$

$\chi^2 = \frac{(n-1)s^2}{a}$

a = hypothesized value

REPEATED-MEASURES ANOVA

$SS_{between} = \Sigma \frac{T^2}{n} - \frac{G^2}{N}$ $df_{between} = k - 1$

$SS_{error} = SS_{within} - SS_{subjects}$

$df_{error} = (N - k) - (n - 1)$

where $SS_{within} = \Sigma SS_{\text{inside each treatment}}$

and $SS_{subjects} = \Sigma \frac{P^2}{k} - \frac{G^2}{N}$

$F = \frac{MS_{between}}{MS_{error}}$ where each $MS = \frac{SS}{df}$

$H_0: \sigma_1^2 = \sigma_2^2$ (independent)

$F_{n_1 - 1 | n_2 - 1}$

$F = \frac{s_1^2}{s_2^2}$

turn over for $H_0: \sigma_1^2 = \sigma_2^2$ (dependent)

one tailed z w/ $\alpha = 0.05$ is	1.645		
two tailed z	1.96	$\times\ \sigma_m$ = critical value ± the mean	
two tailed z w/ $\alpha = 0.01$	2.58		
one tailed z	2.33		

$df_{\text{between treatments}}$ = number of cells − 1

$SS_{\text{within treatments}} = \Sigma SS_{\text{each treatment}}$

$df_{\text{within treatments}} = \Sigma df_{\text{each treatment}}$

$SS_A = \Sigma \frac{T_{ROW}^2}{n_{ROW}} - \frac{G^2}{N}$

df_A = (number of levels of A) − 1

$SS_B = \Sigma \frac{T_{COL}^2}{n_{COL}} - \frac{G^2}{N}$

df_B = (number of levels of B) − 1

$SS_{A \times B} = SS_{\text{between treatments}} - SS_A - S$

$df_{A \times B} = df_{\text{between treatments}} - df_A - df_B$

$F_A = \frac{MS_A}{MS_{within}}$ $F_B = \frac{MS_B}{MS_{within}}$

where each $MS = \frac{SS}{df}$

PEARSON CORRELATION

$r = \frac{SP}{\sqrt{SS_X SS_Y}}$

where $SP = \Sigma(X - M_X)(Y - M_Y) = \Sigma$

SPEARMAN CORRELATION

$r_s = 1 - \frac{6\Sigma D^2}{n(n^2 - 1)}$

LINEAR REGRESSION

$\hat{Y} = bX + a$ where $b = \frac{SP}{SS_X}$ an

$SS_{regression} = r^2 SS_Y$ $df = 1$ MS_{regre}

$SS_{residual} = (1 - r^2)SS_Y$ $df = n - 2$

MULTIPLE REGRESSION

$\hat{Y} = b_1 X_1 + b_2 X_2 + a$

$R^2 = \frac{b_1 SP_{X1Y} + b_2 SP_{X2Y}}{}$

$H_0: \sigma_1^2 = \sigma_2^2$ (dependent)

$$t = \frac{s_1^2 - s_2^2}{\sqrt{\frac{4 s_1^2 \cdot s_2^2}{n-2}(1 - r_{1,2}^2)}}$$

Hypothesis w/correlations

$H_0: \rho_{xy} = a$

$$z = \frac{Z_r - Z_a}{1/\sqrt{n-3}}$$

$H_0: \rho_{xy} = 0$

$$t(n-2) = \frac{r_{xy}}{\sqrt{(1-r^2)/n-2}}$$

$H_0: \rho_1 = \rho_2$ (independent)

$$z = \frac{Z_{r_1} - Z_{r_2}}{\sqrt{\frac{1}{n_1 - 3} + \frac{1}{n_2 - 3}}}$$

Cohen's d = $\frac{\text{mean difference}}{SD}$ =

$\frac{M - \mu}{S}$

$\boxed{r^2 = \frac{t^2}{t^2 + df}}$

To determine power for a one tailed test:

1. calculate $\sigma_m = \frac{\sigma}{\sqrt{n}}$
2. find critical value by multipling the z score at the alpha level for the test by σ_m Then add or subtract that value from $\mu_{true} = X$
3. $\frac{X - \mu_{hypo}}{\sigma_m} = z$
4. Look-up the portion in the tail in the z table that is the power

For a two tailed test

1. calculate $\sigma_m = \frac{\sigma}{\sqrt{n}}$
2. Same as step two above but both add + subtract to get two critical values
3. Calculate two z scores for each critical value
4. Same as above but look-up both z scores
5. Add the two values found in step 4

*Not necessarily the portion in the tail draw the two distributions to make sure of the value!

Repeated Measures

ANOVA

Source	SS	df	MS	F
Between treatments (btw)	$\sum \frac{T^2}{n} - \frac{G^2}{N}$ for each cell (same as all other ANOVAS in the book)	$k-1$	$\frac{SS_{btw}}{df_{btw}} =$ MS_{btw}	$F = \frac{MS_{btw}}{MS_{error}}$ $df = (df_{btw}, df_{error})$
Within treatments broken into	$SS_{w/i} = \sum SS$ the sum of all these values inside each cell	$N-K$		
→ Btw. Subjects	$\sum \frac{P^2}{k} - \frac{G^2}{N}$ P = Person Score	$n-1$		
→ Error Term	$SS_{w/i} - SS_{btw\_subs}$	$(N-k)-(n-1)$ (work backwards to find)	$\frac{SS_{error}}{df_{error}} =$ MS_{error}	
Note	$SS_{btw.\,subjects}$ + SS_{error} should = $SS_{w/i}$ treatments	$df_{btw\,subjects}$ + $df_{error} \stackrel{?}{=} df_{w/i}$		
total	$SS_{btw} + SS_{w/i}$	$df_{btw} + df_{w/i}$		

Two factor ANOVA

G = Grand Sum = $T_{row_1} + T_{row_2}$ or for 2x2 $T_{col_1} + T_{col_2}$

T = The sum of the scores in each cell

Source	SS	df	MS	F
Between treatments (btw)	$\sum \frac{T^2}{n} - \frac{G^2}{N}$ ↓ For each cell	# of cells − 1	don't calculate	don't calculate
Factor A	$\sum \frac{T_{row}^2}{n_{row}} - \frac{G^2}{N}$	# of rows − 1	$MS_A = \frac{SS_A}{df_A}$	$F_A = \frac{MS_A}{MS\ within}$ $(df_A, df_{w/i})$
Factor B	$\sum \frac{T_{col}^2}{n_{col}} - \frac{G^2}{N}$	# of columns − 1	$MS_B = \frac{SS_B}{df_B}$	$F_B = \frac{MS_B}{MS\ within}$ $(df_B, df_{w/i})$
A×B interaction	$SS_{btw} - SS_A - SS_B$	$df_{A \times B} = df_{btw} - df_a$ also $- df_b$	$MS_{A \times B} = \frac{SS_{A \times B}}{df_{A \times B}}$	$F_{A \times B} = \frac{MS_{A \times B}}{MS\ within}$ $(df_{A \times B}, df_{w/i})$
Note	$SS_A + SS_B + SS_{A \times B}$ should $= SS_{btw}$	$df_A + df_B + df_{A \times B}$ should $= df_{btw}$		
Within Treatments (w/i)	$SS_{w/i} = \sum SS$ the sum of all the SS inside each treatment cell	$df_{w/i} = \sum df$ each treatment; $df_{e/t} = n - 1$ e/t = each treatment for each cell	$MS_{w/i} = \frac{SS_{w/i}}{df_{w/i}}$	N/A
Total	$\sum X^2 - \frac{G^2}{N}$ or $SS_{btw.} + SS_{w/i}$	df total = $df_{btw} + df_{w/i}$	N/A	N/A

also $df_{A \times B} = df_A \times df_B$

See also figure 15.6 pg. 507
for df see unit 2 slides pg. 12

Scheffé

for unequal n's or making complex comparisons

$$S_K = \sqrt{MS_{error} \cdot \left(\frac{a_A^2}{n_A} + \frac{a_B^2}{n_B} + \ldots \frac{a_K^2}{n_K}\right)}$$

a_A = contrast coefficient of the mean of subgroup A

n_A = # of cases in subgroup A

$S = (F')(S_K)$ F' = formula on otherside

Fisher's LSD

if equal n's

$$lsd = \sqrt{\alpha F(1, df_{error})} \cdot \sqrt{\frac{(2) \cdot MS_{Error}}{n}}$$

for unequal n's or making complex comparisons

$$lsd = \sqrt{\alpha F(1, df_{error})} \cdot \sqrt{MS_{error} \cdot \left(\frac{a_A^2}{n_A} + \frac{a_B^2}{n_B} + \ldots \frac{a_K^2}{n_K}\right)}$$

$\alpha F(1, df_{error})$ = F value at the selected alpha level w/df of 1 + the df of MS_{error} (MS_{within})

$$SS_{Total} = \sum X^2 - \frac{G^2}{N} \text{ or } SS_{within} + SS_{between}$$

N = total number scores

$$SS_{within} = \sum SS_{\text{inside Each Treatment}}$$

or

$$SS_{within} = \sum X^2 - \sum \frac{T^2}{n}$$

T = total sum of scores for each treatment

n = number of scores in each treatment

$$SS_{between} = \sum \frac{T^2}{n} - \frac{G^2}{N}$$

G = Sum of all scores (grand sum)

$df_{total} = N-1$

$df_{within} = N-K$ or $\sum(n-1)$

$df_{between} = K-1$

K = number of treatment condition or levels

$$F = \frac{MS_{Between}}{MS_{within}} = \frac{SS_{btw}/df_{btw}}{SS_{wi}/df_{wi}}$$

$$SS = \sum X^2 - \frac{(\sum X)^2}{N}$$

$$\eta^2 = \frac{SS_{between}}{SS_{total}} \text{ (one-way)}$$

Tukey HSD

For equal n's

$$HSD = q\sqrt{\frac{MS_{Error}}{n}}$$

q = number of levels + df for error term

For unequal n's

$$HSD = q \cdot \sqrt{\frac{MS_{Error}}{2}} \cdot \sqrt{\frac{1}{n_i} + \frac{1}{n_j}}$$

Scheffé

equal n's $S = (F')(S_K)$

$$F' = \sqrt{(K-1) \cdot F_{criterion}}$$

$$S_K = \sqrt{\frac{(2) \cdot MS_{error}}{n}}$$

$F_{criterion}$ = critical value for F

K = # of groups

$MS_{error} = MS_{within}$

n = # in each individual treatment

Two factor ANOVA (

G = Grand Sum = $T_{row_1} + T_{row_2}$ or for 2x2 $T_{col_1} + T_{col_2}$

T = The sum of the scores in each cell

Source	SS	df	MS	F
Between treatments (btw)	$\sum \frac{T^2}{n} - \frac{G^2}{N}$ ↓ For each cell	# of cells − 1	don't calculate	don't calculate
Factor A	$\sum \frac{T^2_{row}}{n_{row}} - \frac{G^2}{N}$	# of rows − 1	$MS_A = \frac{SS_A}{df_A}$	$F_A = \frac{MS_A}{MS_{within}}$ $(df_A, df_{w/i})$
Factor B	$\sum \frac{T^2_{col}}{n_{col}} - \frac{G^2}{N}$	# of columns − 1	$MS_B = \frac{SS_B}{df_B}$	$F_B = \frac{MS_B}{MS_{within}}$ $(df_B, df_{w/i})$
A×B interaction	$SS_{btw} - SS_A - SS_B$	$df_{AxB} = df_{btw} - df_a$ also $- df_b$	$MS_{AxB} = \frac{SS_{AxB}}{df_{AxB}}$	$F_{AxB} = \frac{MS_{AxB}}{MS_{within}}$ $(df_{AxB}, df_{w/i})$
Note	$SS_A + SS_B + SS_{AxB}$ should = SS_{btw}	$df_A + df_B + df_{AxB}$ should = df_{btw}		
Within Treatments (w/i)	$SS_{w/i} = \sum SS$ the sum of all the SS inside each treatment cell	$df_{w/i} = \sum df_{each\ treatment}$ $df_{e/t} = n - 1$ e/t = each treatment for each cell	$MS_{w/i} = \frac{SS_{w/i}}{df_{w/i}}$	N/A
Total	$\sum X^2 - \frac{G^2}{N}$ or $SS_{btw.} + SS_{w/i}$	df total = $df_{btw} + df_{w/i}$	N/A	N/A

also $df_{AxB} = df_A \times df_B$

See also figure 15.6 pg. 507
for df see unit 2 slides pg. 12